ELSEVIER

Physica B 263–264 (1999) 37–39

PHYSICA B

Diffusive transport of acoustic waves in strongly scattering media

J.H. Page[a,*], I.P. Jones[a], H.P. Schriemer[a], M.L. Cowan[a], Ping Sheng[b], D.A. Weitz[c]

[a]*Department of Physics and Astronomy, University of Manitoba, Winnipeg, MB, Canada R3T2N2*
[b]*Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong*
[c]*Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104-6396, USA*

Abstract

The diffusive transport of multiply scattered ultrasonic waves is investigated experimentally and theoretically in a simple system consisting of glass beads in water. New experimental results are presented using a novel method for measuring the frequency correlation function of the transmitted acoustic field. The wave diffusion coefficient D is found to vary strongly with frequency when the wavelength is comparable to the size of the scatterers, reflecting a substantial slowing down of wave propagation when the scattering is strongest. The results are interpreted using a model based on a spectral function approach that gives good agreement with experiment.

Keywords: Acoustic wave diffusion; Multiple scattering of ultrasound

The propagation of classical waves through strongly scattering media [1,2] is a problem of considerable current interest in many areas of physics, where important questions remain to be answered on topics as diverse as in the investigation of classical wave localization and the development of novel non-destructive probes of disordered materials. Much of the recent progress in these areas has been facilitated by the use of the diffusion approximation, in which the propagation of the multiply scattered waves is treated as a random walk process; in this approximation, the transport of energy is governed by the transport mean free path l^* (the average distance traveled by the waves before their direction of propagation becomes randomized), the energy velocity v_E (the local velocity of energy transport between random walk steps) and the diffusion coefficient $D = v_E l^*/3$. In this paper, we investigate the diffusive propagation of ultrasonic waves through a simple strongly scattering medium consisting of glass beads immersed in water. We present a novel way of measuring the wave diffusion coefficient, and discuss its frequency dependence in the intermediate frequency regime where the ultrasonic wavelength is comparable with the size of the scatterers. Because of the simplicity of the scattering medium investigated in these experiments, we are able to interpret our results

* Corresponding author: Tel: +1-204-474-9852; fax: +1-204-474-7622; e-mail: jhpage@cc.umanitoba.ca.

0921-4526/99/$ – see front matter
PII: S0921-4526(98)01191-0

quantitatively using a model based on a spectral function approach, giving new insight into wave transport in this strongly scattering regime [3].

The experiments were performed on disk-shaped samples containing 1 mm diameter glass beads packed in water at a volume fraction of about 63%. The cells were placed in a water tank between a generating transducer, operating in the low MHz frequency range, and a miniature hydrophone detector, which measured the transmitted ultrasonic field in a single coherence area or speckle spot [4]. Fig. 1(a) shows a typical transmitted waveform when a short incident pulse is used, illustrating the interference of the multiply scattered signals that have followed different scattering paths through the sample. In our previous experiments [3,4], we determined the diffusion coefficient of the multiply scattered ultrasound from the time dependence of the average transmitted intensity, measured by ensemble averaging the square of the envelope of the scattered waveforms over statistically independent speckles. Here we follow a different approach, in which we determine D using an unconventional method for measuring the frequency correlation function of the multiply scattered acoustic waves. In contrast to the traditional continuous-wave methods used in light scattering, we have developed a pulsed technique in which we use a short incident pulse, thus ensuring that the sample is probed simultaneously at a wide range of frequencies. This enables us to determine the frequency-dependent response function of the scattering medium $R(\nu)$ by calculating the ratio of the complex Fast Fourier Transforms (FFTs) of the scattered $S(\nu)$ and input $I(\nu)$ waveforms, $R(\nu) = S(\nu)/I(\nu)$. The response function for one of the speckles is illustrated in Fig. 1(b), showing the range of frequencies probed in this particular experiment. The frequency correlation function $g_1(\Delta\nu)$ is calculated directly from $R(\nu)$ as $g_1(\Delta\nu) = \langle R^*(\nu)R(\nu+\Delta\nu)\rangle/\langle R^2\rangle$, and $g_1(\Delta\nu)$ is then ensemble averaged over many ($\sim$100) speckles. Correlation functions measured over the frequency range from 2 to 3 MHz are shown in Fig. 2 for several sample thicknesses L. Physically, the fluctuations with frequency in $R(\nu)$ and the corresponding decay of the correlation function result from the change in the interference pattern in a single speckle as the number of wavelengths contained along all the multiple scattering paths increases. Within the diffusion approximation, the average path length is L^2/l^* ; since $g_1(\Delta\nu_{1/2}) \sim \frac{1}{2}$ when the average path length contains one additional wavelength, this implies that $\Delta\nu_{1/2} \sim 3D/L^2$, showing in simple physical terms why D can be determined from $g_1(\Delta\nu)$.

To determine D accurately from $g_1(\Delta\nu)$ [5], it is necessary to account quantitatively for the effects of both absorption and boundary reflections, as

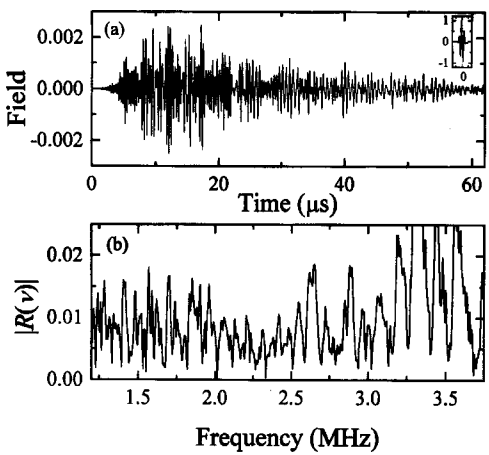

Fig. 1. (a) Time dependence of the transmitted ultrasonic field in a single speckle for the short input pulse shown in the insert, and (b) the corresponding frequency-dependent response function $R(\nu)$. The sample thickness was = 10.2 mm.

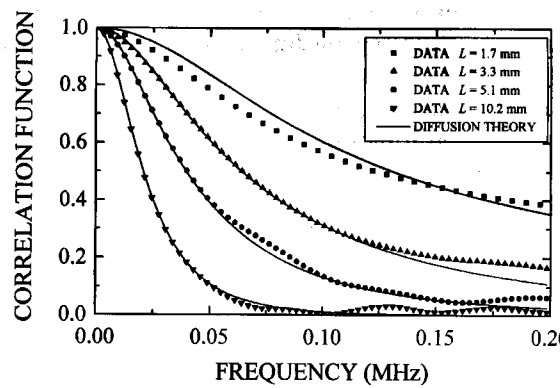

Fig. 2. Magnitude of the frequency correlation function over the frequency interval from 2–3 MHz for several sample thicknesses L. The solid curves are fits to diffusion theory with $D = 0.4\ \mathrm{mm^2/\mu s}$ and $\tau_a = 12\ \mu s$.

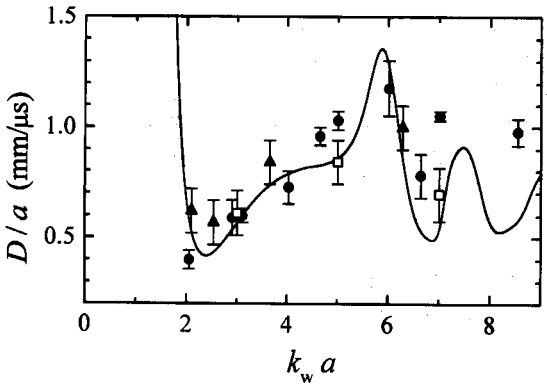

Fig. 3. Diffusion coefficient, normalized by the bead radius a, as a function of the normalized frequency $k_w a$, where k_w is the ultrasonic wavevector in water. The frequency correlation data are represented by the open symbols, time domain data by the solid symbols and the predictions of our theoretical model by the solid curve.

discussed in Ref. [4] for the analysis of the diffuse intensity time profile. The solid curves in Fig. 2 show the results of these calculations, from which we determine both D and the absorption time τ_a. Excellent agreement with diffusion theory is found for the three thickest samples where $L \geqslant 4l^*$, indicating that diffusion theory can be reliably used for acoustic waves down to rather small L. However, clear discrepancies between theory and experiment are evident for the thinnest sample ($L \sim 2l^*$), where the diffusion approximation breaks down even for the relatively long paths that are probed by $g_1(\Delta\nu)$ as $\Delta\nu \to 0$.

Fig. 3 compares our frequency correlation measurements of D (open symbols) with the time-domain data measured previously [3] (solid symbols). Considerable dispersion is found, with D varying by approximately a factor of 3 over the frequency range investigated. To explain these results, we have developed a model, based on a spectral function approach, that allows us to determine $D = v_E l^*/3$ directly from calculations of v_E and l^*, starting from an accurate description of the ballistic propagation of the weak coherent component that is not scattered out of the forward direction [6]. Thus we find that the low values of D near $k_w a \sim 2$ are due to extremely slow values of v_E, the origin of this strong dispersion being the scattering delay experienced by a wave pulse as it undergoes many successive scatterings in the sample [3]. Our predictions for D are shown by solid curve in Fig. 3, and are in good agreement with our data over an extended range of frequencies. These results give a simple physical picture of energy transport by diffusive acoustic waves in strongly scattering media – an important step in facilitating both the search for acoustic wave localization in more strongly scattering samples and the development of new dynamic phonon scattering techniques such as Diffusing Acoustic Wave Spectroscopy [5,7,8].

References

[1] P. Sheng (Ed.), Scattering and Localization of Classical Waves in Random Media, World Scientific, Singapore, 1990.
[2] P. Sheng, Introduction to Wave Scattering, Localization, and Mesoscopic Phenomena, Academic Press, San Diego, 1995.
[3] H.P. Schriemer, M.L. Cowan, J.H. Page, P. Sheng, Z. Liu, D.A. Weitz, Phys. Rev. Lett. 79 (1997) 3166.
[4] J.H. Page, H.P. Schriemer, A.E. Bailey, D.A. Weitz, Phys. Rev. E 52 (1995) 3106.
[5] I.P. Jones, M.Sc. Thesis, University of Manitoba, 1996.
[6] J.H. Page, P. Sheng, H.P. Schriemer, I. Jones, X. Jing, D.A. Wietz, Science 271 (1996) 634.
[7] I.P. Jones, J.H. Page, D.A. Weitz, Phys. Canada 52 (1996) 129.
[8] M.L. Cowan, I.P. Jones, J.H. Page, D.A. Weitz, to be published.